Quantitative Literacy:
Thinking Between the Lines

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Chapter 3:
Linear and Exponential Change:

Comparing Growth Rates

Chapter 3: Linear and Exponential Changes
Lesson Plan

- Lines and linear growth: What does a constant rate mean?
- Exponential growth and decay: Constant percentage rates
- Logarithmic phenomena: Compressed scales

Learning Objectives:

- Understand linear functions and consequences of a constant growth rate.
- Interpret linear functions.
- Calculate and interpret the slope.
- Understand linear data and trend lines for linear approximations.

3.1 Lines and linear growth: What does a constant rate mean?

- A **linear function** is a function with a constant growth rate.
- A **graph** of a linear function is a straight line.
- **Example (Determining linear or not):** Find the growth rate of the function. Make a graph of the function. Is the function linear?

  For my daughter's wedding reception, I pay $500 rent for the building plus $15 for each guest. This describes the total cost of the reception as a function of the number of guests.

- **Solution:** The growth rate is the extra cost incurred for each additional guest, that is $15. So, the growth rate is **constant**.

  The additional cost means each additional guest.

  The total cost of the reception is a **linear** function of the number of guests.

3.1 Lines and linear growth: What does a constant rate mean?

- **Example (Determining linear or not):** Find the growth rate of the function and give its practical meaning. Make a graph of the function. Is the function linear?

  My salary is initially $30,000, and I get a 10% salary raise each year for several years. This describes my salary as a function of time.

- **Solution:** The growth rate:

$$1^{\text{st}} \text{ year increased} = 10\% \text{ of } \$30{,}000 = \$3{,}000$$

$$\therefore \quad 1^{\text{st}} \text{ year salary} = \$33{,}000$$

$$2^{\text{nd}} \text{ year increased} = 10\% \text{ of } \$33{,}000 = \$3{,}300$$

$$\therefore \quad 2^{\text{nd}} \text{ year salary} = \$36{,}300$$

The growth rate is **not the same each year**. So, the graph is **not** a straight line. Thus, my salary is **not** a linear function of time in years.

Chapter 3 Linear and Exponential Changes

3.1 Lines and linear growth: What does a constant rate mean?

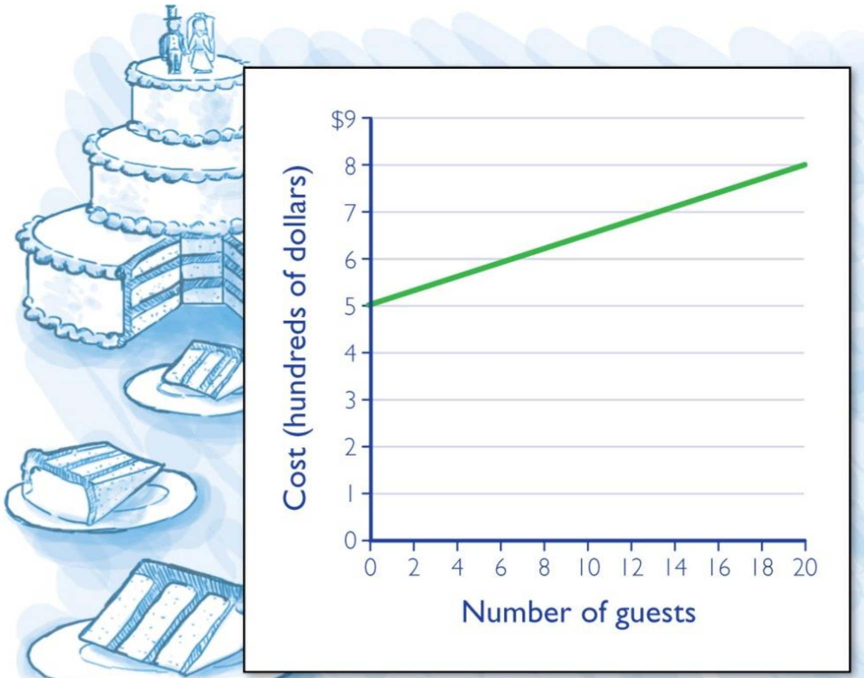

FIGURE 3.5 Cost is a linear function of number of wedding guests.

FIGURE 3.6 Salary is not a linear function of time.

3.1 Lines and linear growth: What does a constant rate mean?

Formula for Linear Function

$$y = \text{Growth rate } \times x + \text{Initial value}$$

If *m* is the growth rate or slope and *b* is the initial value,

$$y = mx + b.$$

- **Example:** Let L denote the length in meters of the winning long jump in the early years of the modern Olympic Games. Suppose L is a function of the number n of Olympic Games since 1990, an approximate linear formula is $L = 0.14n + 7.20$.

  Identify the initial values and growth rate, and explain in practical terms their meaning.

- **Solution:** The initial value is 7.20 meters. The growth rate is 0.14 meter per Olympic Game. It means that the length of the winning long jump increased by 0.14 meters from one game to the next.

3.1 Lines and linear growth: What does a constant rate mean?

- **Example:** A rocket starting from an orbit 30,000 kilometers (km) above the surface of Earth blasts off and flies at a constant speed of 1000 km per hour away from Earth.

1. Explain why the function giving the rocket's distance from Earth in terms of time is linear.
2. Identify the initial value and growth rate.
3. Find a linear formula for the distance.

The Saturn V carried the first men to the moon in 1969.

- **Solution:**

1. We first choose letters to represent the function and variable. Let d be the distance in km from Earth after t hours.

   The growth rate = velocity = 1000 km/hour = a constant

   Thus, d is a linear function of t.

2. The Initial value = 30,000 km

   = the height above Earth at blastoff

3. d = Growth rate $\times\, t$ + Initial value

   $= 1000\,t + 30{,}000$

Interpreting and using the Slope

- The slope of a linear function is using:

$$\text{Slope} = \text{Growth rate} = \frac{\text{Change in function}}{\text{Change in independent variable}}$$

- Write the equation of a linear function as: $y = mx + b$, the formula for the slope becomes

$$m = \text{Slope} = \frac{\text{Change in } y}{\text{Change in } x}$$

- Each 1-unit increase in *x* corresponds to a change of *m* units in *y*.

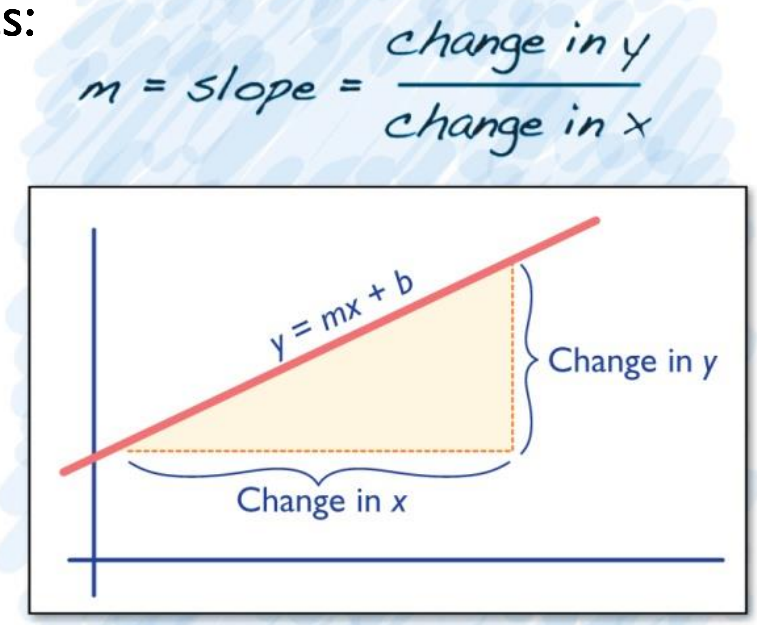

FIGURE 3.9 How slope is calculated.

3.1 Lines and linear growth: What does a constant rate mean?

- **Example:** Suppose your car's 20-gallon tank is full when you begin a road trip.

  Assume that you are using gas at a constant rate, so the amount of gas in your tank is a linear function of the time in hours you have been driving.

  After traveling for two hours, your fuel gauge reads three-quarters full.

  Find the slope of the linear function and explain in practical terms what it means.

3.1 Lines and linear growth: What does a constant rate mean?

- **Solution:**

- When the tank is ¾ full, there are 15 gallons of gas left in the tank.
- The amount of gas in the tank has decreased by 5 gallons.
- The change in gas is –5 gallons over 2 hours driving.

$$m = \frac{\text{Change in gas}}{\text{Change in time}} = \frac{-5 \text{ gallons}}{2 \text{ hours}} = -2.5 \text{ gallons per hour}$$

- This means we are using 2.5 gallons of gas each hour.

- **Example:** Measuring temperature using the Fahrenheit scale is common in the United States, but use of the Celsius scale is more common in most other countries.

  The temperature in degrees Fahrenheit (°F) is a linear function of the temperature in degrees Celsius (°C).

  On the Celsius scale, 0°C is the freezing temperature of water. This occurs at 32°F on the Fahrenheit scale. Also, 100°C is the boiling point of water. This occurs at 212°F.

3.1 Lines and linear growth: What does a constant rate mean?

- **Example (cont.):**

1. What is the slope of the linear function giving the temperature in degrees Fahrenheit in terms of the temperature in degrees Celsius?
2. Choose variable and function names, and find a linear formula that converts degrees Celsius to degrees Fahrenheit. Make a graph of the linear function.
3. A news report released by Reuters on March 19, 2002, said that the Antarctic peninsula had warmed by 36°F over the past half-century. The British writer saw a report that the temperature had increased by 2.2°C. Verify that a temperature of 2.2°C is about 36°F.
4. What increase in Fahrenheit temperature should the writer have reported in part 3?

- **Solution:**

1. To find a slope:

   An increase on the Celsius scale from 0 to 100 degrees corresponds to an increase on the Fahrenheit scale from 32 to 212 degrees.

$$\text{slope} = \frac{\text{Change in degrees Fahrenheit}}{\text{Change in degrees Celsius}} = \frac{180^{o}\text{F}}{100^{o}\text{C}} = 1.8^{o}\text{F}/^{o}\text{C}$$

Thus, a 1-degree increase on the Celsius scale corresponds to a 1.8-degree increase on the Fahrenheit scale.

3.1 Lines and linear growth: What does a constant rate mean?

- **Solution:**

2. Use *F* for the temperature in degrees Fahrenheit and *C* for the temperature in degrees Celsius.

   The linear relation we expressed by the formula:

$$F = \text{Slope} \times C + \text{Initial value}$$

$$F = 1.8C + 32$$

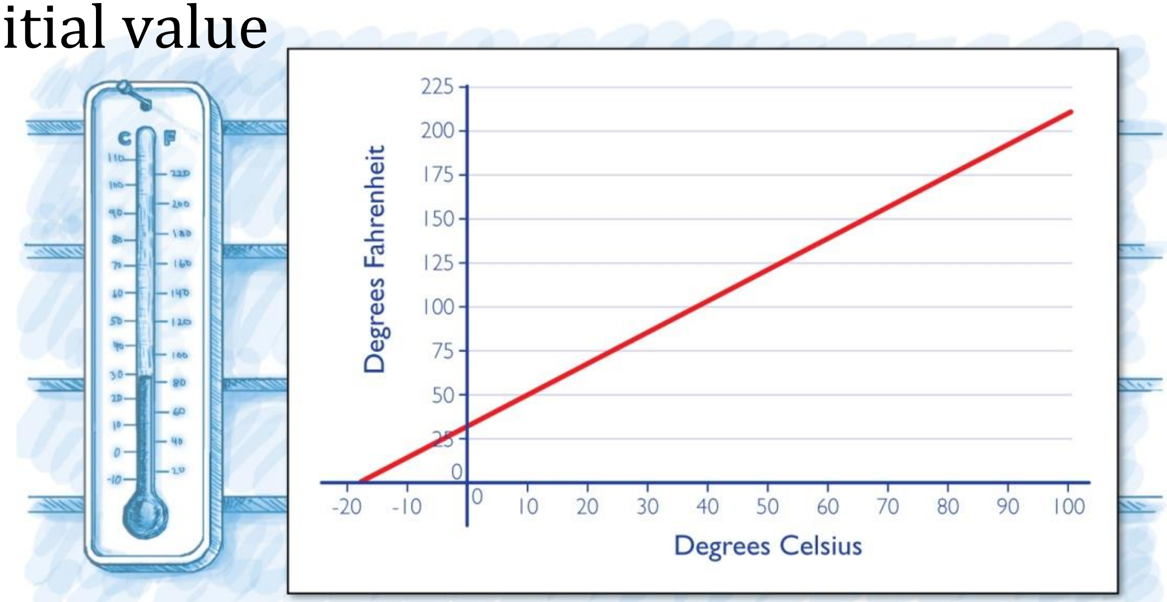

FIGURE 3.11 Fahrenheit temperature is a linear function of Celsius temperature.

- **Solution:**

3. Verify that a temperature of 2.2 degrees Celsius is about 36 degrees Fahrenheit.

   We put 2.2 degrees Celsius into the formula

   $$\mathrm{F} = \mathrm{Slope} \times \mathrm{C} + \mathrm{Initial\ value}$$

   to convert Celsius to Fahrenheit:

   $$\begin{aligned} \mathrm{F} &= 1.8\,\mathrm{C} + 32 \\ &= (1.8 \times 2.2) + 32 \\ &= 35.96^{\mathrm{o}}\mathrm{F} \end{aligned}$$

- **Solution:**

4. What increase in Fahrenheit temperature should the writer have reported in part 3?

   The slope of the linear function is $1.8^{\text{o}}\text{F per }^{\text{o}}\text{C}$.

   An increase of 2.2^{o}C corresponds to an increase of

   $$2.2 \times 1.8 = 3.96^{\text{o}}\text{F}.$$

   The writer should have reported a warming of about 4^{o}F.

3.1 Lines and linear growth: What does a constant rate mean?

- Given a set of data points, the **regression line** (or **trend line**) is a line that comes as close as possible to fitting those data.
- **Example:** The following table shows the running speed of various animals vs. their length. Show the scatterplot and find the formula for the trend line. Explain in practical terms the meaning of the slope.

Animal	Length (inches)	Speed (feet per second)
Deer mouse	3.5	8.2
Chipmunk	6.3	15.7
Desert crested lizard	9.4	24.0
Grey squirrel	9.8	24.9
Red fox	24.0	65.6
Cheetah	47.0	95.1

3.1 Lines and linear growth: What does a constant rate mean?

- **Solution:**

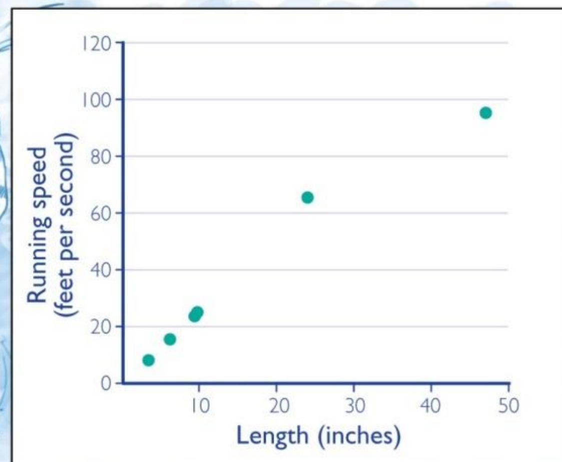

FIGURE 3.14 Scatterplot of running speed versus length.

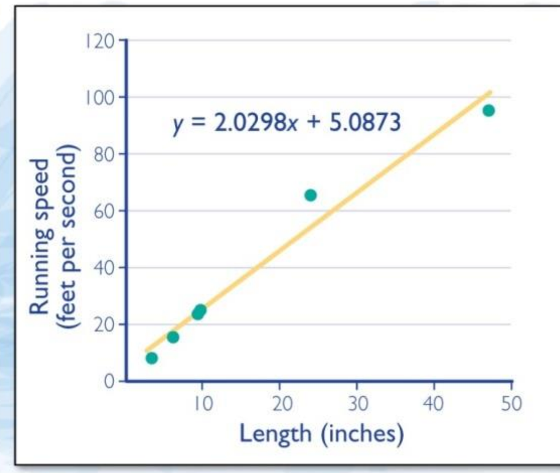

FIGURE 3.15 Trend line added.

The points do not fall on a straight line, so the data in the table are not exactly linear. In Figure 3.15, we have added the trend line produced by the spreadsheet program Excel.

- **Solution:** The equation of the trend line is:

$$y = 2.03x + 5.09$$

This means that running speed *S* in feet per second can be closely estimated by:

$$S = 2.03\,L + 5.09,$$

where *L* is the length measured in inches.

The slope of the trend line is 2.03 feet per second per inch.

This value for the slope means that an animal that is 1 inch longer than another would be expected to run about 2.03 feet per second faster.

Chapter 3 Linear and Exponential Changes:
Chapter Summary

- **Lines and linear growth:** What does a constant rate mean?
  - Understand linear functions and consequences of a constant growth rate.

    Recognizing and solve linear functions

    Calculate the growth rate or slope

    Interpolating and using the slope

    Approximate the linear data with trend lines

Chapter Summary

- **Exponential growth and decay:** Constant percentage rates
  - Understand exponential functions and consequences of constant percentage change.

    The nature of exponential growth

    Formula for exponential functions

    The rapidity of exponential growth

    Relating percentage growth and base

    Exponential decay

    Radioactive decay and half-life

Chapter Summary

- **Logarithmic phenomena:** Compressed scales
  - Understand the use if logarithms in compressed scales and solving exponential equations.

    The Richter scale and interpolating change on the Richter scale

    The decibel as a measure of sound

    Solving exponential equations

    Doubling time and more